Formation of density waves in traffic flow through intersecting roads

B. Ray and S. N. Bhattacharyya
Department of Mechanical Engineering, Indian Institute of Technology, Kharagpur 721302, West Bengal, India
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The formation of density waves in two intersecting roads, with a traffic circle at the intersection, is studied. It is found that, depending on the traffic densities in the two roads, density waves can form in the traffic circle and in one or both of the roads. Depending on the expression chosen for the optimal velocity, either the congestion moves entirely to the traffic circle or the congestion becomes confined to the traffic circle and a part of the road approaching the traffic circle.

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I. INTRODUCTION

Traffic flow has been studied using a variety of methods. These can be broadly classified into macroscopic and microscopic models. One of the earliest studies [1] considered a macroscopic fluid model for the flow of traffic. This model predicted the existence of density waves in traffic flow and also showed that traffic jams can form by a mechanism similar to the formation of shock waves in a compressible fluid [2]. This study showed that density waves can travel either forward or backward along the road depending on the traffic density. Since 1990, with rapidly increasing availability of computing power, a number of microscopic models for traffic flow have been suggested and extensive studies have been carried out using these models. Prominent among these are the car following models [3,4], cellular automaton model [5], and the gas kinetic model [6]. Car following models include follow-the-leader model [3] and the optimal velocity model [4]. Using the optimal velocity model, again density waves have been studied. It was shown that these waves grow from a small initial disturbance when the density of traffic exceeds a certain threshold value [4]. Further simulation showed that jamming transition occurs when this threshold is crossed.

Many of the studies on traffic flow consider only one road. This is appropriate for modeling highway traffic but not for city traffic. For city traffic a network of intersecting roads would appear to be more appropriate. For such a configuration most of the studies [7–15] use the cellular automaton model. In the pioneering study of Biham *et al.* [7], which gave the model now known as the Biham-Middleton-Levine (BML) model, it was shown that when the traffic density is larger than a critical value, after a finite time, the system reaches a jammed state where all vehicles come to a halt. However, it is recognized that this model makes some rather simplifying assumptions. In the original model [7] each cell represented a traffic intersection and there is effectively no modeling of the traffic flow on the road between two adjacent intersections. In order to remove this shortcoming more realistic models have been proposed which include additional cells between the intersections [16,17]. However, these models still have the feature that two vehicles at an intersection can bring traffic to a halt at that intersection. This behavior can change qualitatively when modifications are made in the model. When the vehicles are located on the bonds and not at the intersections it is found [18] that jams form but have finite lifetime and consequently permanent jamming as in the BML model does not occur. A more realistic cellular automaton model for traffic flow, which takes into account the length of the road, has two lanes on each road for traffic in opposite directions, and provides rotaries at the intersections, shows that, depending on the rules to be followed by the vehicles at the rotary, the system can exhibit a variety of phenomenon [19]. It would, therefore, appear to be of interest to use some of the other models which have been used for studying highway traffic and see what these predict for city traffic. Accordingly, in this study we will use the optimal velocity model, which provides a fairly accurate representation of a vehicle moving on a road, to study the behavior of traffic in intersecting roads.

While a lot of studies have been carried out on density waves in highway traffic [4,20] there seem to be no studies on density waves in intersecting roads. For studying density waves it would appear that the optimal velocity model or a fluid model would be more appropriate than a cellular automaton model. This is further supported by the observation that while pedestrian flow has been studied extensively using the cellular automaton model [20] the existence of density waves in pedestrian flow was brought out in a recent study which used an optimal velocity model for pedestrian flow [21]. The difficulty in using a fluid model for traffic in intersecting roads is that we do not have a way of modeling road intersections in a fluid model for traffic flow. Therefore, we will use the optimal velocity model. We now have to provide a model for the street intersection. At the intersection traffic can be regulated either by traffic lights or by a traffic circle or a roundabout. While a lot of studies on traffic flow in the presence of traffic lights are available [22–29] this is expected to produce intermittent rather than continuous traffic flow and would, therefore, not be suitable for studying density waves. There is a recent study of traffic flow in the presence of a roundabout [30]. Traffic circles are similar to roundabouts but are usually larger and the rules governing entry of vehicles can also be different. In our study we will assume that a traffic circle is present at the intersection. At the traffic circle we will use a different logic from that used for the roundabout in Ref. [30] in order to study density waves in intersecting roads. We plan to address the following questions. Where do density waves first set in, in the traffic circle or in one of the roads? If the traffic density is different in the two roads, can density waves in the road with higher

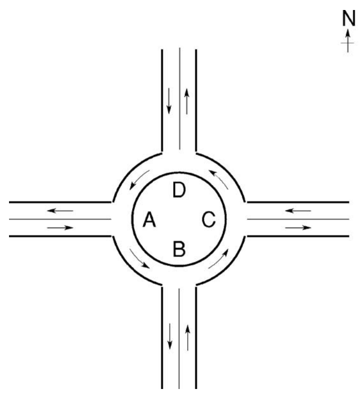

FIG. 1. Two intersecting roads with a traffic circle at the intersection.

traffic density cause forced density waves to occur in the road with lower density? Similarly can density waves propagate from the traffic circle to the roads? The plan of the paper is as follows. In Sec. II we describe the configuration that we will study and define the rules for traffic regulation at the traffic circle, and in Sec. III we present the results obtained and discuss our findings, while in Sec. IV we provide a short conclusion.

II. FORMULATION

In the earliest study of density waves using the optimal velocity model [4] a very simple configuration was considered—viz., only one road carrying traffic in only one direction with the ends closed by assuming periodic boundary conditions. In order to study density waves in intersecting roads, we will similarly choose the simplest configuration which contains a street intersection and can support density waves. We consider two roads which intersect as shown in Fig. 1, with a traffic circle at the intersection. For both roads we will assume periodic boundary conditions. One way to justify the periodic boundary conditions is to assume that both the roads are circular as shown in Fig. 2. Here we assume that a traffic circle is present at intersection point R while at S there is a two-level crossing. Therefore, traffic in the two roads interact only at the point R. We assume that both the east-west and north-south roads have separate lanes for traffic moving in opposite directions but

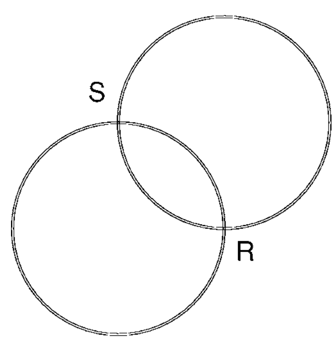

FIG. 2. Configuration of two intersecting roads which satisfy the periodic boundary conditions.

the traffic circle, which has vehicles moving in one direction only, has only one lane. We assume that there is traffic moving in all four directions but there is no turning of vehicles so that any vehicle coming to the traffic circle goes halfway around it and continues to move in its original direction.

In each of the roads we assume that traffic flow is governed by the optimal velocity model with the vehicle acceleration given by [4,20]

$$\frac{d^2x_n}{dt^2} = a\left\{ V(\Delta x_n) - \frac{dx_n}{dt} \right\}, \tag{1}$$

where

$$\Delta x_n = x_{n+1} - x_n. \tag{2}$$

Here x_n is the coordinate of the nth vehicle, Δx_n is the headway with the vehicle in front which has coordinate x_{n+1}, a is a constant representing the driver's sensitivity, and $V(\Delta x)$, the legal or optimal velocity, is a function of the headway. In Ref. [4] the expression

$$V(\Delta x) = \tanh(\Delta x - h_c) + \tanh(h_c), \tag{3}$$

where h_c is a constant representing the safety distance, was used. The authors considered a "simple model" which assumes $h_c=0$ and a "realistic model" which assumes $h_c=2$. For the simple model, when parameter values are such that density waves can occur, they found that vehicles at times move backwards. In order to overcome this they proposed the realistic model and showed that with this model backward movement of vehicles does not occur. There is still one shortcoming of this model: that the length of the vehicle is not taken into account. So when the density of traffic is high, the headway between vehicles can become smaller than the actual vehicle length. If we assume that a vehicle actually has this length, it would mean a collision. This shortcoming is overcome in a more refined model. Based on observed data from Japanese motorways, the following expression for optimal velocity was suggested [31–33]:

$$V(\Delta x) = 16.8\{\tanh[0.086(\Delta x - 25)] + 0.913\}. \tag{4}$$

While Eq. (3) is in terms of dimensionless variables, in Eq. (4) all quantities are expressed in SI units. When Eq. (4) is used a finite headway is maintained between vehicles. In our study we will use both Eq. (3), which is more widely used, and Eq. (4), which maintains a finite headway and thus prevents collisions from occurring. When we use Eq. (3) for $V(\Delta x)$, following Ref. [4], we assume $a=1$ in Eq. (1), while with Eq. (4) for $V(\Delta x)$ we use $a=2\ \mathrm{s}^{-1}$ as in Ref. [32]. In applying the optimal velocity model to traffic on intersecting roads the difficulty is in computing headway for the lead vehicle approaching an intersection between the traffic circle and one of the roads.

In a recent study [30] simulations were carried out for traffic flow in two intersecting roads with a roundabout at the intersection. There it was assumed that a vehicle approaching a roundabout looks to the quadrant of the roundabout to its left and only if this quadrant is empty it enters the roundabout. The difficulty with this scheme is that if the size of the roundabout is large, this leads to large waiting times for vehicles trying to enter the roundabout. One way of reducing this waiting time is to use the logic that a vehicle trying to enter the roundabout looks for a gap which is larger than the safe distance. If we want vehicles to be able to enter the roundabout without long waiting times, then we require the headway between vehicles in the roundabout to be larger than the safe distance. But this would not be suitable for our study of density waves since density waves occur when headways are of the order of the safe distance. Again traffic circles have rules for entry which are different from those for a roundabout. Some traffic circles require circulating traffic to yield to vehicles entering the traffic circle. Giving priority to entering vehicles, these circles tend to "lock up" with higher traffic volumes. Therefore, we choose a different logic to govern the entry of a vehicle to the traffic circle. We now explain the logic we have used. How this can be implemented in practice is discussed in the last paragraph in Sec. III.

Let us consider the point A in Fig. 1 where the east-bound lane of the east-west road meets the traffic circle. Once one vehicle crosses this point we have to decide whether the lead vehicle in the east-bound lane or the lead vehicle in the quadrant DA of the traffic circle gets priority to enter quadrant AB of the traffic circle. We first check whether there are any vehicles in quadrant DA. If not, the lead vehicle in the east-bound lane gets priority to enter the traffic circle. If there are vehicles in DA, we check whether the lead vehicle is a west-moving vehicle or a south-moving vehicle. If it is a west-moving vehicle, it will go from quadrant DA of the traffic circle to the west bound lane of the east-west road. Therefore, we give priority to the lead vehicle in the east-bound lane to enter quadrant AB of the traffic circle. If the lead vehicle in DA is a south-moving vehicle, at the instant when the previous vehicle crossed A, we compare the distances of the lead vehicle in quadrant DA and in the east-bound lane from the point A. Whichever is nearer is given priority to enter quadrant AB of the traffic circle. Once this priority is set, it is kept unchanged until the vehicle which has priority crosses A. At that instant the same logic is again used to decide which vehicle is to be given priority to enter quadrant AB. Similar logic is used to decide priorities at the points B, C, and D.

Once the priorities have been decided, we can now compute the effective headway for the lead vehicles approaching points A, B, C, and D either from one of the roads or from the traffic circle. For a vehicle which is approaching A from the east-bound lane, if it has priority to enter quadrant AB of the traffic circle and there is at least one vehicle in AB, its headway is the distance from the first vehicle in quadrant AB which is $\Delta x_e + \Delta x_a$, where Δx_e is the distance from the lead vehicle P in the east lane to the intersection point A and Δx_a is the distance from A to the first vehicle Q in quadrant AB, as shown in Fig. 3(a). On the other hand, if it does not have priority, then the lead vehicle in quadrant DA will first enter quadrant AB and then the vehicle from the east-bound lane will follow. If Δx_e is the distance of the lead vehicle P in the east-bound lane from A and Δx_d is the distance of the lead vehicle Q in quadrant DA from, A as shown in Fig. 3(b), and we assume that the two vehicles have approximately the same velocity, then, when the lead vehicle from quadrant DA covers a distance Δx_d and reaches A, the lead vehicle in the east-bound lane would also have moved approximately Δx_d and its distance from point A would be $\Delta x_e - \Delta x_d$. Therefore, the effective headway would be $\Delta x_e - \Delta x_d$. The remaining option is when the lead vehicle in the east-bound lane has priority to enter quadrant AB and there is no vehicle in AB. In this case the effective headway will be with the lead vehicle in the north-bound lane of the north-south road which has priority to enter quadrant BC and is given by $\Delta x_e + L_q - \Delta x_n$, where L_q is the length of a quadrant of the traffic circle, Δx_e is the distance from the lead vehicle P in the east-bound lane to the intersection point A, and Δx_n is the

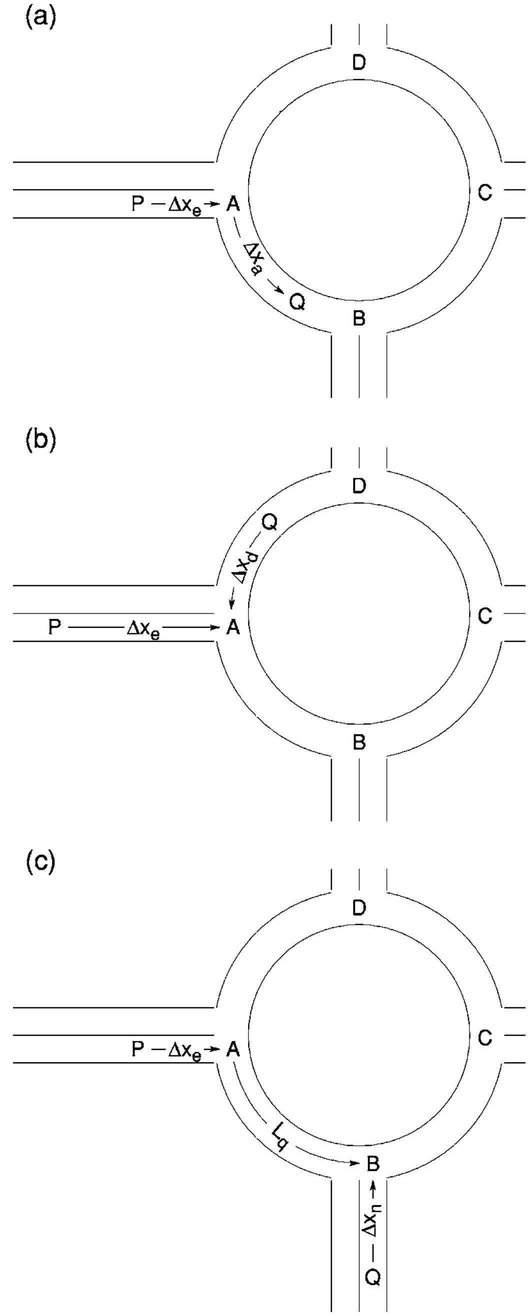

FIG. 3. Effective headway for various possible positions of vehicles approaching an intersection point between a road and the traffic circle.

distance from the lead vehicle Q in the north-bound lane to the intersection point B, as shown in Fig. 3(c). We assume that $\Delta x_e + L_q - \Delta x_n > 0$. For the densities which we will consider the headways are typically much smaller than L_q and, therefore, this condition is usually satisfied. If we consider very low densities, there is a possibility that this condition may not be satisfied. Then the logic for deciding priority and the calculation of headway would have to be appropriately modified. However, in this study we do not consider such low densities.

We now consider the lead vehicle in quadrant AB of the traffic circle. If it is a south-moving vehicle, then its headway will be the distance from the first vehicle in the south-bound lane of the north-south road which is $\Delta x_a + \Delta x_s$ where Δx_a is the distance from the lead vehicle in the quadrant AB to the intersection point B and Δx_s is the distance from B to the first vehicle in the south-bound lane. If it is an east-moving vehicle and has priority to enter quadrant BC of the traffic circle, then the effective headway is $\Delta x_a + \Delta x_b$ or $\Delta x_a + L_q + \Delta x_e$, depending on whether there is or there is not any vehicle in quadrant BC, where Δx_a is the distance from the lead vehicle in quadrant AB to the intersection point B and Δx_b is the distance from B to the first vehicle in quadrant BC if there is one, while Δx_e is the distance from the point C to the first vehicle on the east-bound lane when there is no vehicle in quadrant BC. If the lead vehicle in AB is an east-moving vehicle and does not have priority to enter BC, then the effective headway is $\Delta x_a - \Delta x_n$ where Δx_a and Δx_n are the distances of the lead vehicles in quadrant AB and in the north-bound lane from the intersection point B.

We have used the distance from the intersection point to decide which of the two vehicles approaching the intersection point gets priority. Alternatively, we could have used the criterion that the vehicle which is expected to reach the intersection point earlier should get priority. However, the difficulty is that the effective headway can change abruptly when priority is decided leading to significant change in velocity. So the actual time taken may be quite different from what was predicted using the velocity before priority was decided, though the order in which the vehicles reach the intersection point would probably not change. Moreover, for realistic values of headway we expect velocities of both vehicles to be comparable. If we use Eq. (3) with $h_c = 2$, then in the zero-density limit $\Delta x \to \infty$, we have $V = 1.9640$, while at the onset of instability which corresponds to $\Delta x = 2.8814$ (as shown in Ref. [4]) we have $V = 1.6711$, a decrease of only about 15%.

We assume that the lengths of the east-west and north-south roads are L_{ew} and L_{ns} and the length of each quadrant of the traffic circle, as mentioned earlier, is L_q. We consider N_e, N_w, N_n, and N_s to be the number of east-, west-, north-, and south-moving vehicles some of which, at any particular instant, may be in the traffic circle. To specify the position of any vehicle in any road or quadrant of the traffic circle we use a local coordinate x which measures the distance from the starting point of that part. Thus for an east-moving vehicle x is measured from point C while in quadrant BC of the traffic circle x is measured from point B. To start the simulation we assume that all the vehicles are in the roads and not in the traffic circle. Following Bando *et al.* [4] the east-moving vehicles are assigned initial position and velocity in the east-bound lane:

$$x_1 = \delta,$$

$$x_n = L_{ew}\frac{(n-1)}{N_e}, \quad n = 2, \ldots, N_e,$$

$$\frac{dx_n}{dt} = 0, \quad n = 1, \ldots, N_e, \tag{5}$$

where $\delta = 0.1$ when Eq. (3) is used while $\delta = 1$ when Eq. (4) is used. The N_w, N_n, and N_s vehicles, which are moving west, north, and south, are assigned initial position and velocity in the appropriate lanes in a similar manner. Since there are no vehicles in the traffic circle, the lead vehicle in each lane of the two roads is given priority to enter the traffic circle.

III. RESULTS AND DISCUSSION

We first use Eq. (3) for the optimal velocity and choose $L_{ew} = L_{ns} = 100$ and $L_q = 20$. At first we use the "simple model" of Bando *et al.* [4]; i.e., in Eq. (3) we assume $h_c = 0$. For this model, in the configuration studied in Ref. [4], instability sets in when the headway is less than 0.8814. We assume that $N_e = N_w = N_n = N_s$. For $N_e = \cdots = 53$ the position of the 27th east-moving vehicle versus time is shown in Fig. 4(a). The short segments represent the part spent in quadrants AB and BC of the traffic circle while the longer segments represent the part spent in the east-west road. We observe that in each part the vehicle moves with approximately constant velocity. When we increase the number of vehicles to $N_e = \cdots = 54$ the position versus time of the 27th east-moving vehicle is shown in Fig. 4(b), which clearly shows the presence of density wave in the traffic circle but not in the east-west road. Thus there is a sharp transition to instability when the number of vehicles in each road increases from 53 to 54. For $N_e = \cdots = 80$ the position versus time for the 40th east-moving vehicle is shown in Fig. 4(c). We observe more oscillations in the traffic circle and the vehicle spends more time in the traffic circle but there are no density waves in the east-west road. We have carried out computations for $N_e = \cdots = 150$ and still find that the density waves are confined to the traffic circle. We now compare the value of headway in the traffic circle at which instability sets in with that for the simple model in Ref. [4]. For $N_e = \cdots = 53$ the number of vehicles in quadrant AB of the traffic circle versus time is shown in Fig. 5. We observe that it varies between 15 and 20. For the headway to be 0.8814 the number of vehicles should be approximately 23. Thus our results are in rough agreement with the predictions of Ref. [4] though instability seems to set in at a lower value of traffic density compared with Ref. [4]. The persistence of oscillations in Fig. 5 probably indicates the existence of natural modes in the system.

The shortcoming of the simple model is that, when density waves set in, vehicles can move backwards and this can be clearly seen in Figs. 4(b) and 4(c). To avoid this the "realistic model" was proposed by Bando *et al.* [4]. We now use the realistic model with $h_c = 2$. To begin with we still assume

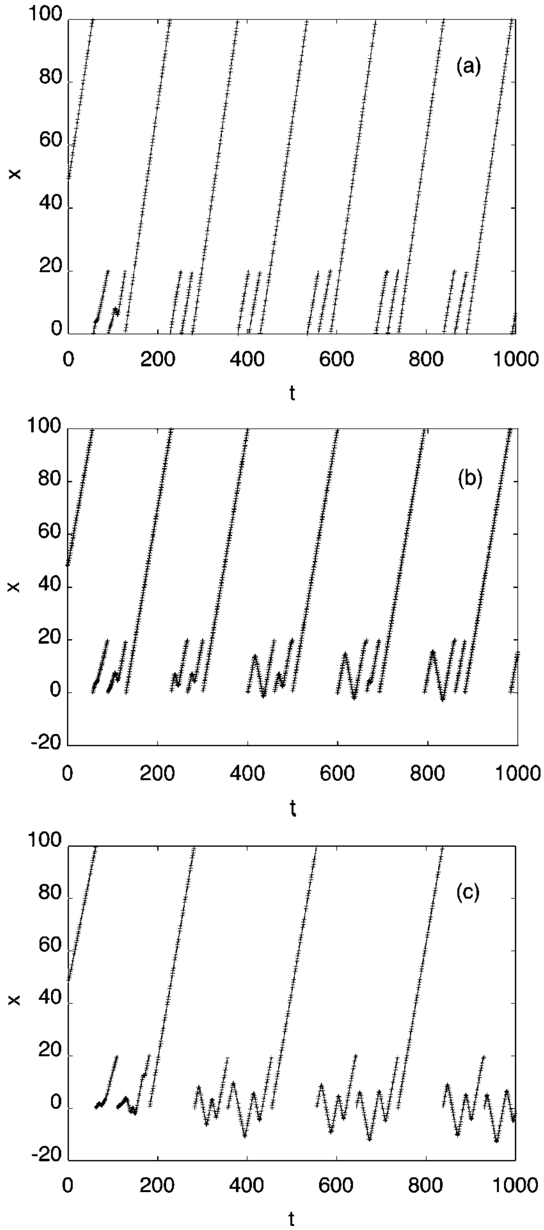

FIG. 4. Trajectories of one east-moving vehicle computed using the simple model with $L_{ew}=L_{ns}=100$ and $L_q=20$. (a) Trajectory of the 27th east-moving vehicle for $N_e=N_w=N_n=N_s=53$, (b) trajectory of the 27th east-moving vehicle for $N_e=N_w=N_n=N_s=54$, and (c) trajectory of the 40th east-moving vehicle for $N_e=N_w=N_n=N_s=80$.

$N_e=N_w=N_n=N_s$. For $N_e=\cdots=10$ the position of the 5th east-moving vehicle is shown in Fig. 6(a) and the position of all vehicles in Fig. 6(b). Almost no congestion is observed. For $N_e=\cdots=20$ the position of the 10th east-moving vehicle and the position of all vehicles are shown in Figs. 7(a) and 7(b). We clearly see the presence of density waves in the traffic circle but hardly any in the roads. Increasing the number of vehicles further to $N_e=\cdots=40$ we observe that initially there is congestion in the roads at the approach to the traffic circle but over time the congestion moves entirely to the traffic circle as shown in Fig. 8(a). Further increase to $N_e=\cdots=60$ shows that the decrease in the region of congestion with time is not smooth as can be seen in Fig. 8(b). The question

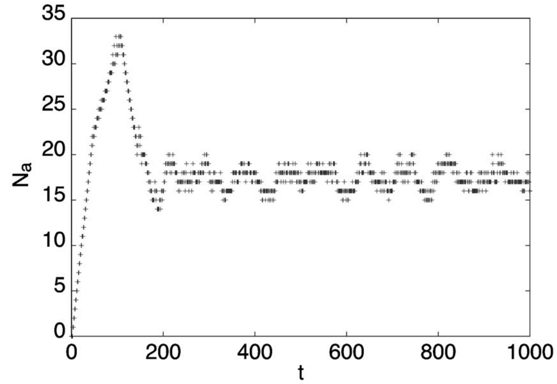

FIG. 5. Number of vehicles in the quadrant AB of the traffic circle, N_a, vs time for traffic governed by the simple model with $L_{ew}=L_{ns}=100$, $L_q=20$, and $N_e=N_w=N_n=N_s=53$.

naturally arises whether the congestion can always move entirely to the traffic circle. Would this still occur if the number of vehicles is larger and the traffic circle is smaller? To test this we consider $L_{ew}=L_{ns}=200$, $L_q=10$, and $N_e=\cdots=100$. As seen in Fig. 9, again the length of the region of congestion in the road keeps decreasing with time. This occurs because we are using Eq. (3) for the optimal velocity so there is no lower limit on the headway between vehicles and the traffic circle

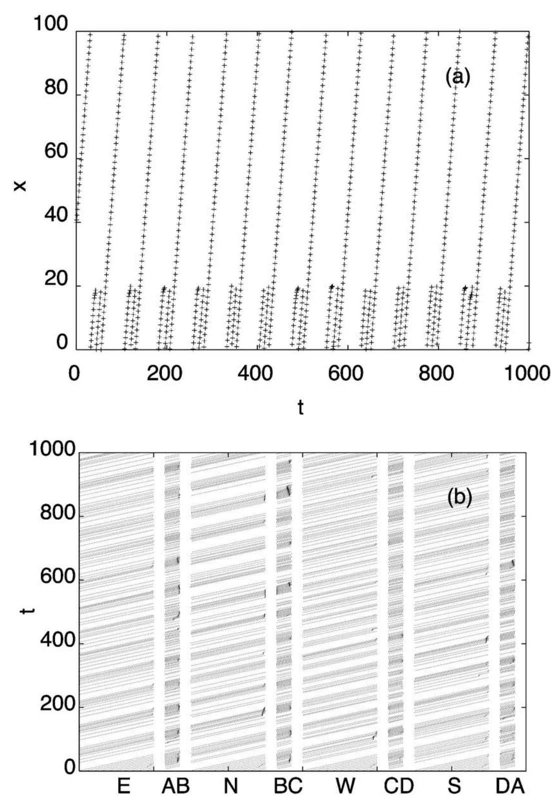

FIG. 6. For traffic governed by the realistic model with $L_{ew}=L_{ns}=100$, $L_q=20$, and $N_e=N_w=N_n=N_s=10$ (a) trajectory of the 5th east-moving vehicle and (b) position of all vehicles vs time. Here "E," "W," "N," and "S" represent the east-, west-, north-, and south-bound lanes and AB, BC, CD, and DA represent the four quadrants of the roundabout.

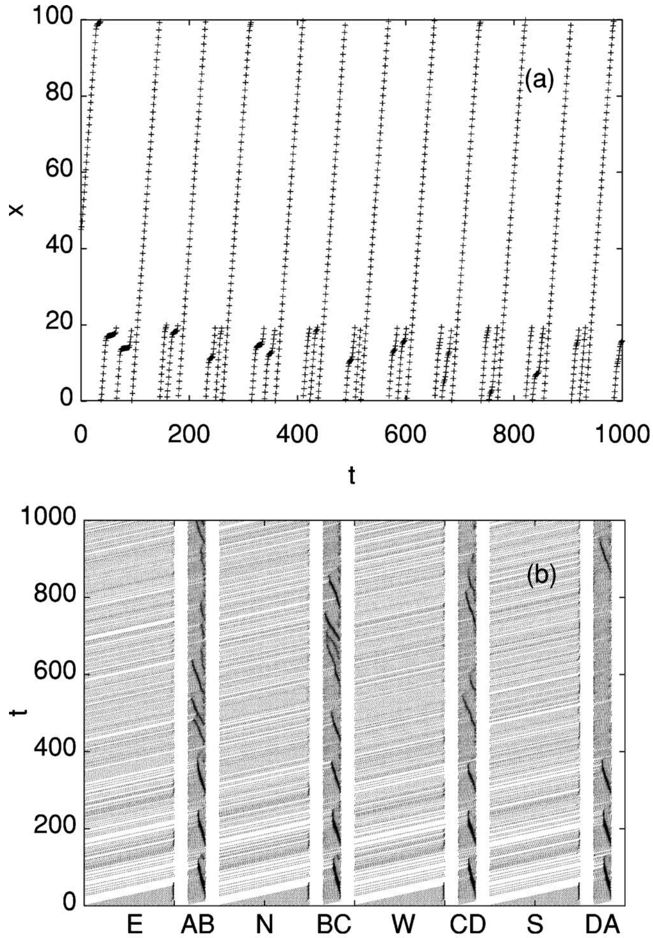

FIG. 7. For $L_{ew}=L_{ns}=100$, $L_q=20$, and $N_e=N_w=N_n=N_s=20$. (a) Trajectory of the 10th east-moving vehicle and (b) position of all vehicles vs time.

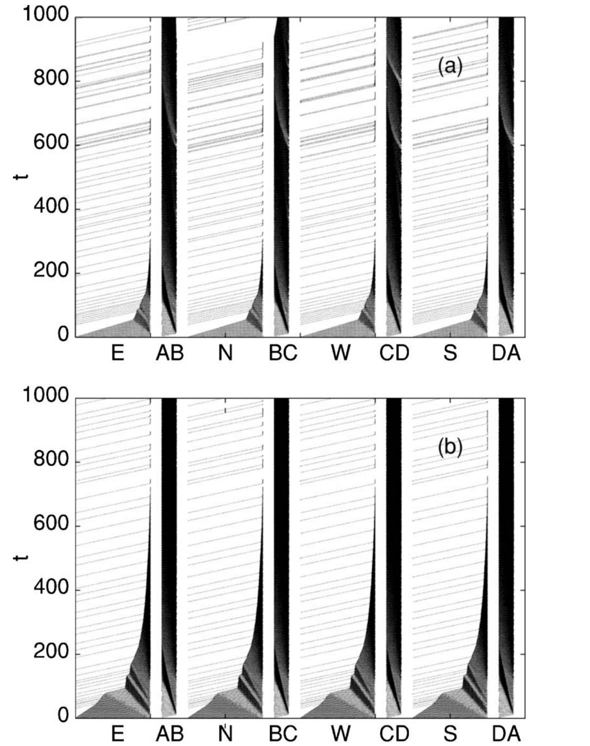

FIG. 8. The position vs time of all vehicles for $L_{ew}=L_{ns}=100$ and $L_q=20$ with (a) $N_e=N_w=N_n=N_s=40$ and (b) $N_e=N_w=N_n=N_s=60$.

can continue to accommodate more congested vehicles. However, this is not realistic. Therefore, in order to maintain a minimum headway we use Eq. (4) for the optimal velocity. For $L_{ew}=L_{ns}=2000$ m, $L_q=200$ m, and $N_e=\cdots=30$ and 60 the positions of all vehicles versus time are shown in Fig. 10. We observe that initially there is congestion over the entire length of the road as well as in the traffic circle but after a short time the congestion is confined to the traffic circle and a part of the road approaching the traffic circle. Furthermore, the width of the region of congestion in the road remains constant with time. This is a significant difference compared to the model given by Eq. (3). Comparing Figs. 10(a) and 10(b) we observe that the width of the region of congestion in the road increases as the number of vehicles is increased. Thus when we use Eq. (3) for the optimal velocity, as we increase the number of vehicles starting from a sufficiently low value we first observe uncongested flow of traffic everywhere; then, density waves begin to appear in the traffic circle and for higher density congestion starts appearing in the roads as well as in the traffic circle. However, if we wait for a sufficient amount of time, the congestion moves entirely to the traffic circle. This remains true even if we increase the number of vehicles and reduce the size of the traffic circle. This can occur because Eq. (3) does not take into account the length of the vehicle and, therefore, any number of congested vehicles can be accommodated in the traffic circle. When we use Eq. (4), which takes into account the length of the vehicle, for a sufficiently large number of vehicles we find that apart from congestion in the traffic circle there is also a region of congestion in the road at the approach to the traffic circle and the width of this region does not decrease with time. In most cases we find that density waves first appear in the traffic circle. This is expected since the vehicle flow rate in each quadrant of the traffic circle is the sum of the flow rate in two directions; e.g., quadrant AB has both south-moving and east-moving vehicles. Density waves have higher amplitude in the traffic

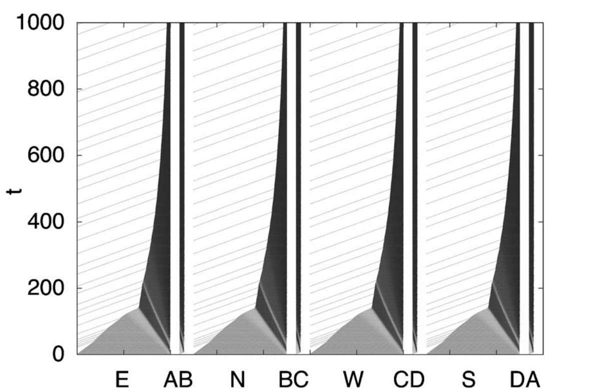

FIG. 9. The position vs time of all vehicles for $L_{ew}=L_{ns}=200$ and $L_q=10$ with $N_e=N_w=N_n=N_s=100$.

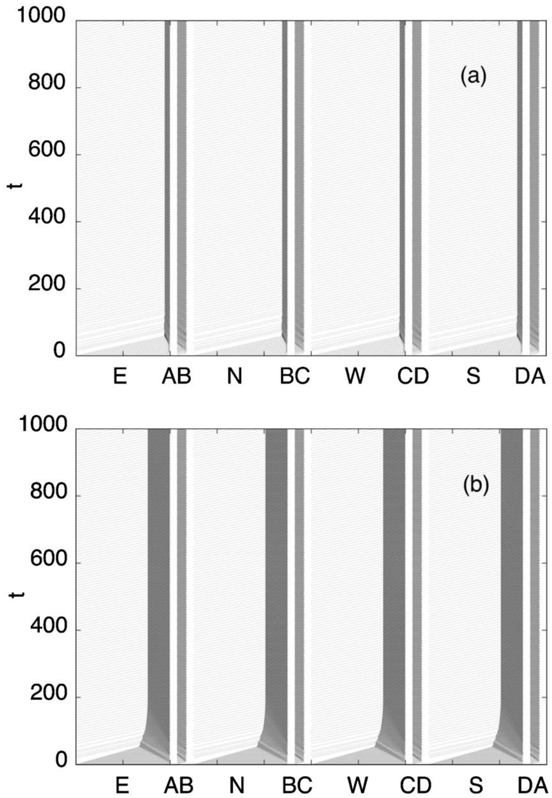

FIG. 10. The position vs time (in seconds) of all vehicles for $L_{ew}=L_{ns}=2000$ m and $L_q=200$ m with (a) $N_e=N_w=N_n=N_s=30$ and (b) $N_e=N_w=N_n=N_s=60$.

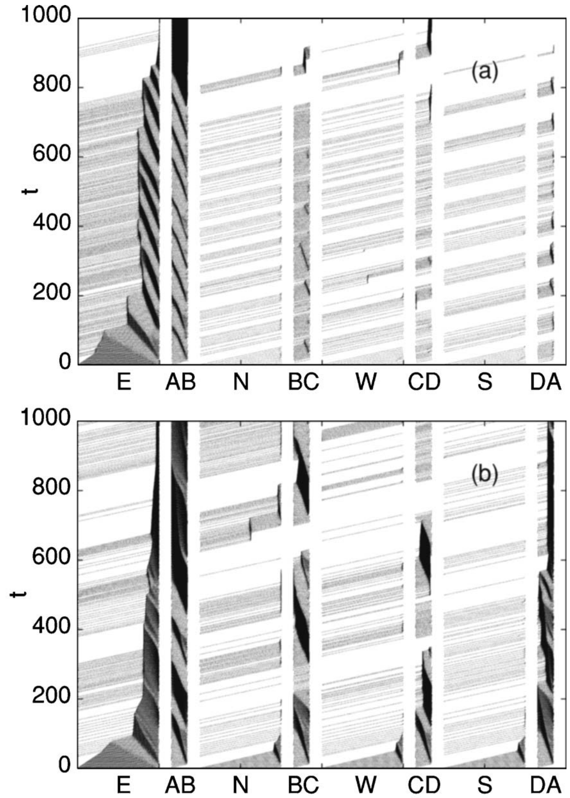

FIG. 11. The position vs time of all vehicles for $L_{ew}=L_{ns}=100$, $L_q=20$, and $N_e=60$ with (a) $N_w=N_n=N_s=10$ and (b) $N_w=N_n=N_s=30$

circle due to higher traffic density in the circle, and this impedes the flow of traffic, leading to a further increase of the density in the traffic circle so that, depending on the expression used for the optimal velocity, the region of congestion becomes confined to the traffic circle or to the traffic circle and a part of the road upstream of the traffic circle.

Next we consider the situation where the number of vehicles moving in different directions are not the same. We first use Eq. (3) for $V(\Delta x)$. For $N_e=60$ and $N_w=N_n=N_s=10$ the positions of all vehicles are shown in Fig. 11(a). We observe congestion in the east-bound lane of the east-west road, in quadrants *AB* and *BC* of the traffic circle and to some extent in the other two quadrants of the traffic circle. For $N_e=60$ and $N_w=N_n=N_s=30$ the position of all vehicles is shown in Fig. 11(b). In this case we observe congestion in the east-bound lane and in all four quadrants of the traffic circle. The larger number of vehicles in the west-, north-, and south-bound lanes causes increase in the number of vehicles in all four quadrants of the traffic circle, and this results in instability and congestion in all four quadrants of the traffic circle. However, in neither case does congestion due to east-moving vehicles cause congestion in the west-, north-, or south-bound lanes. We again observe that over time the congestion moves entirely to the traffic circle. We repeat the calculations using Eq. (4) for $V(\Delta x)$. The positions of all vehicles versus time for $L_{ew}=L_{ns}=2000$ m and $L_q=200$ m with $N_e=45$ and $N_w=N_n=N_s=10$ and 20 are shown in Fig. 12. We observe that with this model a region of congestion remains in the road. Other qualitative features, e.g., in which quadrants of the traffic circle congestion is found to occur, remain the same as obtained using Eq. (3). Thus when the traffic density is different in different directions we observe density waves in some or all quadrants of the traffic circle where the density is high enough. If Eq. (3) is used, after sufficient time no congestion remains in the roads, but if Eq. (4) is used, a region of congestion can remain at the approach to the traffic circle. However, in no instance did we find congestion in the traffic circle or road with high traffic density to propagate into the road with low traffic density. A possible explanation is that any forced wave gets strongly damped in the road with low traffic density which is well below the threshold for instability.

We chose a large size for the traffic circle, $L_q=20$ or 200 m, so that the presence of density waves in the traffic circle could be easily seen. For a smaller traffic circle, $L_q=100$ m, with $L_{ew}=L_{ns}=2000$ m as before, the results are qualitatively similar as seen in Figs. 13(a)–13(c) for $N_e=N_w=N_n=N_s=$ 20, 30, and 40.

As explained earlier, density waves first appear in the traffic circle since the traffic flow rate in any quadrant of the circle is the sum of the traffic flow rate in two directions. This is assuming that there is no turning of the vehicles. The study can be generalized to allow vehicles to turn at the traffic circle. If we assume that the number of vehicles moving in each of the four directions is roughly the same and that vehicles have equal probability of turning left or right, we do

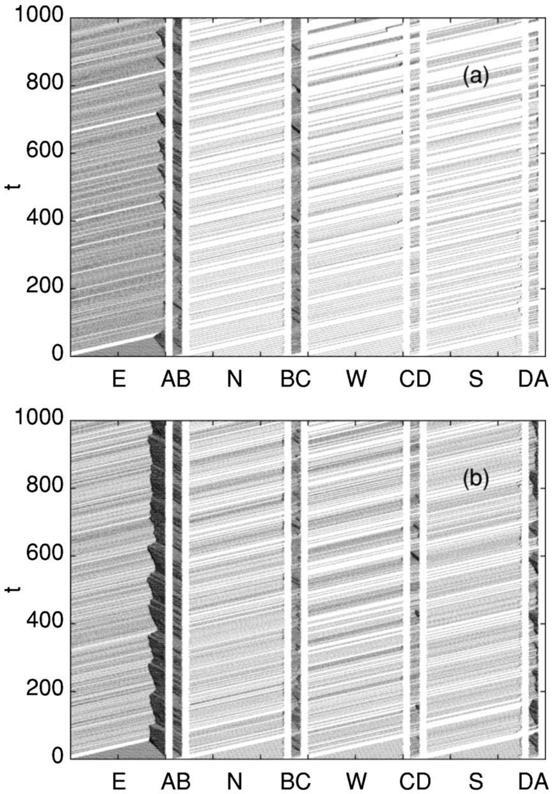

FIG. 12. The position vs time (in seconds) of all vehicles for $L_{ew}=L_{ns}=2000$ m and $L_q=200$ m with (a) $N_e=45$ and $N_w=N_n=N_s=10$ and (b) $N_e=45$ and $N_w=N_n=N_s=20$.

not expect the results to be significantly different since on the average each vehicle would traverse two quadrants of the circle so that for an equal number of vehicles moving in each direction the vehicle flow rate in the circle would be twice the flow rate in any direction in one of the roads. If the number of vehicles moving in different directions are unequal or the probabilities of turning left or right are not the same, we still expect the traffic density in the circle to be higher than in the roads and consequently density waves to form when the traffic density in any quadrant exceeds the threshold for instability.

If we wish to design the system so that it can support higher traffic density without the formation of density waves in the traffic circle, then, from the reasoning given above, we must provide a multilane traffic circle with the number of lanes in the traffic circle equal to twice the number of lanes for traffic in any direction in each of the roads. Thus in the present study we would require a two-lane traffic circle. However, the rules for traffic regulation, which now must also provide for lane changing in the traffic circle, would be necessarily more complex and implementing such logic in practice may not be easy. However, a feasibility study could be of interest and this can form the subject of a future study.

We have assumed that, at the circle, traffic is regulated by a system of assigning priorities to approaching vehicles. With this arrangement we find that until the onset of density waves there is smooth flow of traffic without significant waiting time at the circle. At this point it may be questioned

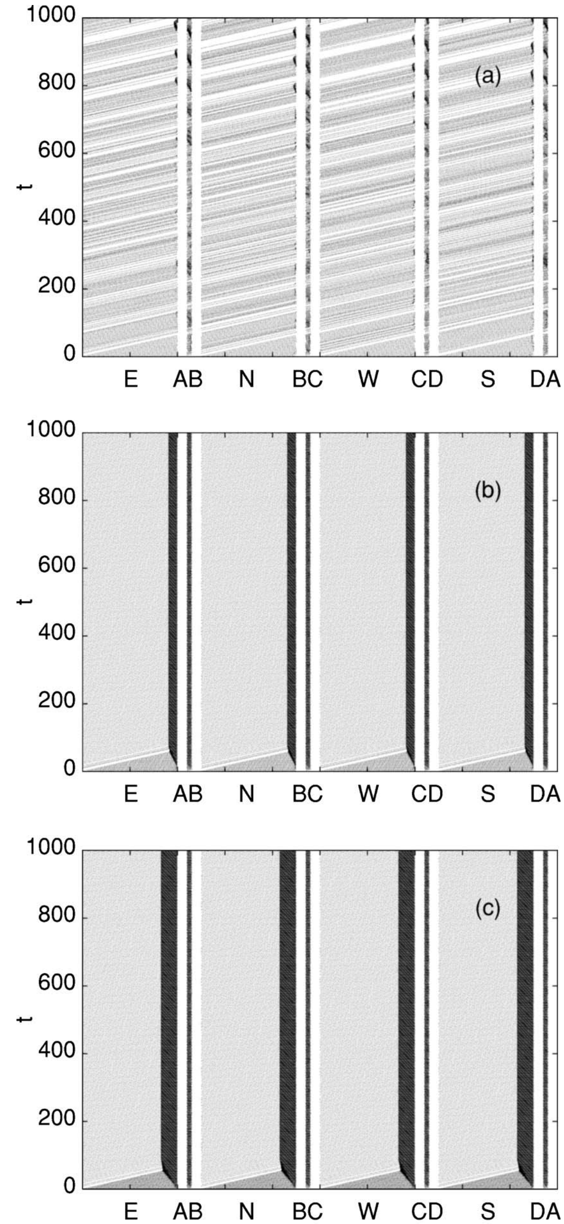

FIG. 13. For $L_{ew}=L_{ns}=2000$ m and $L_q=100$ m the position of all vehicles vs time (in seconds) for (a) $N_e=N_w=N_n=N_s=20$, (b) $N_e=N_w=N_n=N_s=30$, and (c) $N_e=N_w=N_n=N_s=40$.

whether this method for traffic regulation at the traffic circle can be implemented in practice. At present there is serious thinking about automated highways which it is believed can support a vehicle flow rate 3 times larger than present-day highways [34]. If we extend the automation to intersecting roads, the system could be based on the logic used in present study. Alternatively, for manually driven vehicles, we can have sensors to detect the position of vehicles near the traffic circle. This information would be sent to a microprocessor which would be programmed to assign priority based on the logic used in present study. It would then send this information to the vehicles so that the driver of every vehicle would know whether he has priority to proceed.

IV. CONCLUSIONS

We now summarize our findings. For traffic flow in two intersecting roads with a traffic circle at the intersection we find that density waves first appear in those regions where the traffic density is higher than the threshold for instability, usually first in the traffic circle, but for higher density in the roads also. If the optimal velocity model does not take into account the length of the vehicle, then the congestion moves entirely to the traffic circle, but for a model which takes this into account, apart from congestion in the traffic circle, a region of congestion also remains in the road at the approach to the traffic circle. When the number of vehicles moving in different directions are different, density waves which form in a lane with high traffic density are not found to cause forced density waves in a lane with traffic density below the threshold for instability.

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